PILOT LEARNING
CALCULUS II ENGINEERING
PROBLEM-SET 3
FALL 2019

(1) Check that the differential equation $y' + 2y = 2e^x$ is satisfied by the function $y = \frac{2}{3}e^x + e^{-2x}$.

(2) Consider the differential equation $y' = x + y^2$.
   (a) Sketch the direction field of the differential equation.
   (b) Then use it to sketch a solution curve that passes through the point (0,0).

(3) (a) Verify that all members of the family $y = (c - x^2)^{-1/2}$ are solutions of the differential equation $y' = xy^3$.
   (b) What can you say about the graph of a solution of the equation $y' = xy^3$ when x is close to 0? When x is large?
   (c) Find a solution to the initial value problem (IVP)

$$y' = xy^3 \quad ; \quad y(0) = 2$$

(4) Psychologists interested in learning theory study learning curves. A learning curve is the graph of a function $P(t)$, the performance of someone learning a skill as a function of the training time t. The derivative dP/dt represents the rate at which performance improves.
   (a) Based on your own learning experience, sketch what you think a typical learning curve looks like.
      (i) When do you think P increases most rapidly?
      (ii) What happens to dP/dt as t increases?
      (iii) Explain your graph.
   (b) If M is the maximum level of performance of which the learner is capable, explain why the differential equation

$$\frac{dP}{dt} = k(M - P)$$

   where k is a positive constant, is a reasonable model for learning.
   (c) Make a rough sketch of a possible solution of this differential equation. How does this graph compare to the one that you drew in (a)?

(5) Consider the differential equation $y' = -y^2$.
   (a) What can you say about a solution of the equation just by looking at the differential equation?
   (b) Verify that all members of the family $y = 1/(x+C)$ are solutions of the equation in part (a).
   (c) Can you think of a solution of the differential equation $y' = -y^2$ that is not a member of the family in part (b).
   d. Find a solution of the initial-value problem

$$y' = -y^2 \quad ; \quad y(0) = 0.5$$